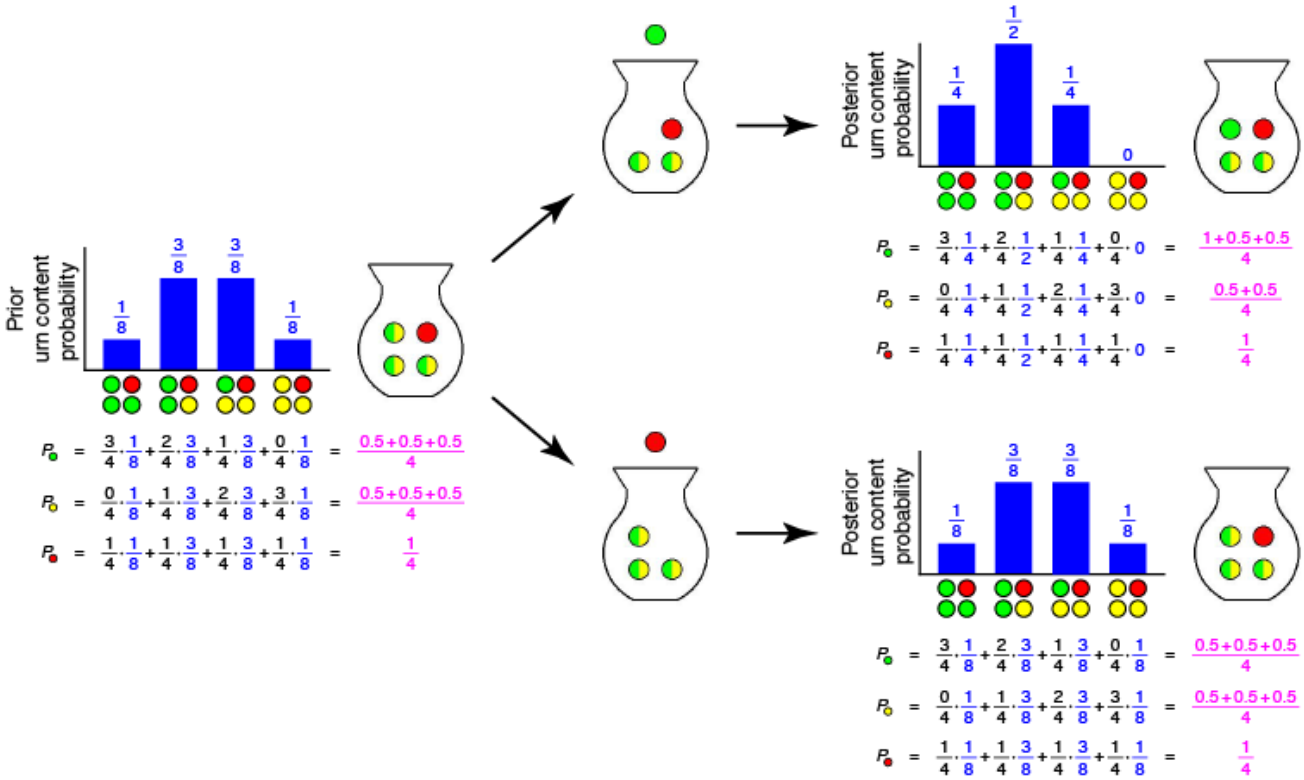
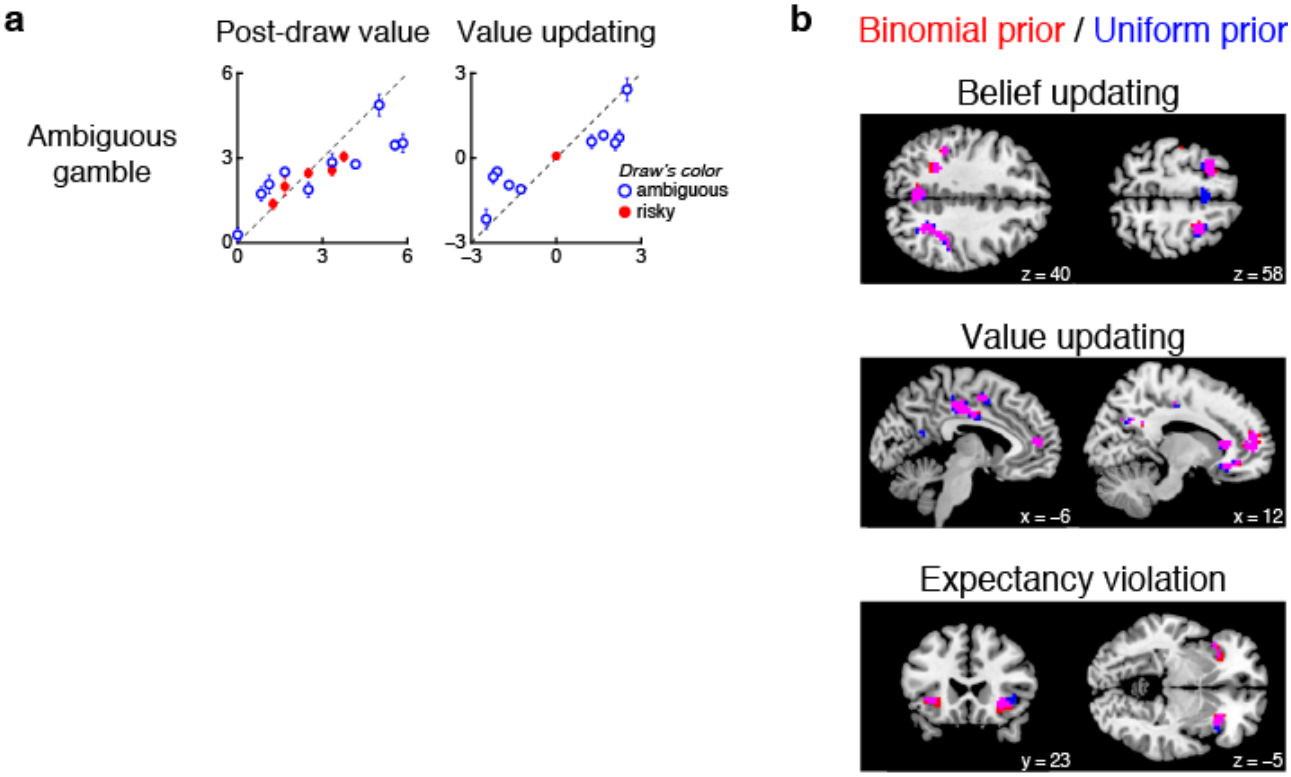
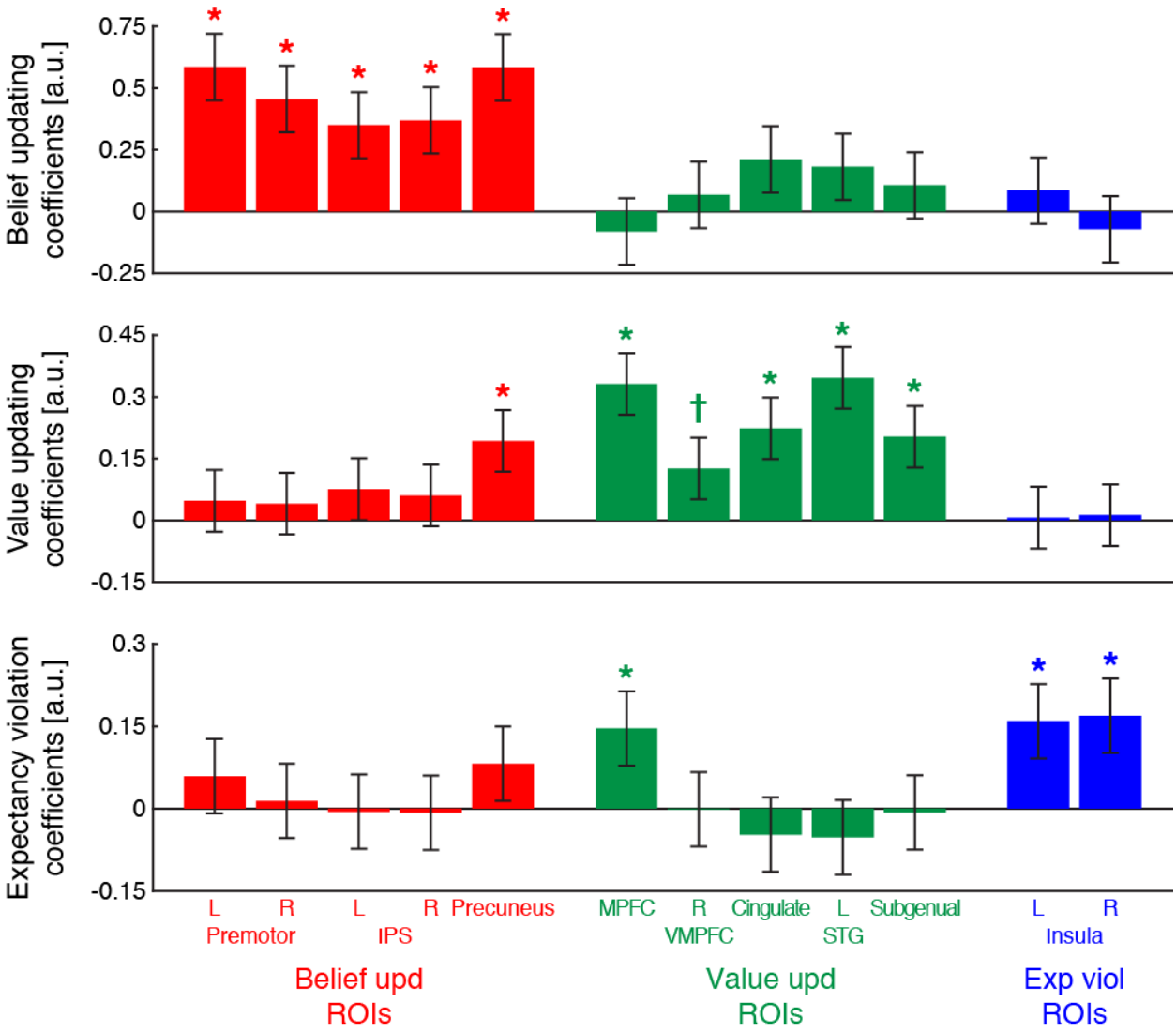
Supplementary Figure 1: Bayesian model of belief updating. Point estimates of probability of a future draw from the urn are generated in belief. To do so, second-order probability over all possible urn contents is constructed and updated according to Bayes' rule (*blue*). Shown is an exemplar urn with four possible contents (*left*). An ambiguous-color draw updates the second-order probability, eliminating the possibility of one possible urn content (*top right*). A risky-color draw does not update the prior (*bottom right*). This Bayesian account is actually mathematically equivalent to a heuristic account, which only considers "effective" urn content by treating each ambiguous ball as a pair of half (0.5) balls (*magenta*). See Supplementary Text for formal descriptions of models.



Supplementary Figure 2: Results based on uniform prior. The results shown in the main text were obtained using the Bayesian model with second-order binomial prior. we also conducted the same analyses with uniform prior to confirm their robustness. (**a**) Behavioral results. Predictions from the uniform model differ from the binomial (original) model only in post-draw value and value updating in ambiguous gambles with ambiguous-color draws. Prediction of the uniform model was successful ($p < .05$, respectively), but worse than the binomial model ($p < .05$; see Supplementary Text). Error bars: standard errors. (**b**) fMRI results. Activation maps of belief updating, value updating, and expectancy violation were quite similar whether we used binomial prior (*red*, the same as Fig. 3d) or uniform prior (*blue*, overlaps in *magenta*). Shown are clusters that survived voxel-wise threshold $p < .001$ (uncorrected) and cluster-size threshold $k > 10$.



Supplementary Figure 3: ROI-wise analysis results. Fig. 3e aggregated activations from all ROIs in the same categories (belief updating, value updating, or expectancy violation). This figure shows the results of the same analysis, but conducted on ROI-wise basis instead. *Top:* Belief updating was correlated with activation in all of the belief-updating ROIs ($p < .05$), but none of value-updating ROIs or expectancy-violation ROIs ($p > .10$). *Middle*: Value updating was correlated with activation in all of the value-updating ROIs ($p < .05$; $p < .10$ in right VMPFC) and a belief-updating ROI in precuneus ($p < .05$), but not in the rest of the belief-updating ROIs or any of the expectancy-violation ROIs ($p > .10$). Note that the precuneus cluster was close to a value-updating ROI in cingulate cortex (Table 2). *Bottom*: Expectancy violation was correlated with activation in both of the expectancy-violation ROIs ($p < .05$) as well as a value-updating ROI in MPFC ($p < .05$), but not in the rest of the value-updating ROIs or any of the belief-updating ROIs ($p > .10$). Error bars: SEMs, asterisks: $p < .05$, daggers: $p < .10$.



Supplementary Text: Bayesian modeling of updating and its alternatives.

In our Bayesian modeling, we postulated that point estimates of a future draw's color probability are calculated in belief. We specifically assumed that all possible urn contents are considered, weighted according to their "second-order" probability, and averaged. Second-order probability over urn contents is updated in a full Bayesian manner, and its prior is binomially distributed.

1. Heuristic account

Even though our behavioral (Fig. 2) and fMRI (Fig. 3) results were derived based on the Bayesian modeling, they are not dependent on specifics of this model. Specifically, predictions of this model are mathematically equivalent to another, more heuristic account, which considers only one "effective" urn content. In this account, each ambiguous ball in the urn (the ball which color could be one of the two ambiguous colors) is treated as a pair of half (0.5) balls in ambiguous colors. When an ambiguous-color draw is observed, one of such pairs is replaced with a full ball in the draw's color. Importantly, this heuristic account does not necessarily require that updating processes are full-Bayesian. Below is the proof of their mathematical equivalence.

Let γ, υ, and ρ be the numbers of green, yellow, and red balls in the urn respectively. Assume that the ambiguous colors are green and yellow, and the risky color is red. Thus, although subjects do not know γ and υ, they know $\gamma + \upsilon$ $(= \alpha)$ and ρ (the total number of the balls in the urn is $\alpha + \rho$). Let g (G), y (Y), and r (R) be the event of the observed (resolution) draw in green, yellow, and red, respectively.

Prior. In the Bayesian model, prior second-order probability over urn contents $(\gamma, \upsilon, \rho) = (\gamma, \alpha - \gamma, \rho)$ follows a binomial distribution:

$$P(\gamma, \upsilon, \rho) = \frac{1}{2^\alpha}\binom{\alpha}{\gamma} \text{ for all } \gamma \text{ s.t. } 0 \le \gamma \le \alpha.$$

Prior probability of a future draw in green is, using binomial theorem,

$$P(g) = \sum_\gamma P(g|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho) = \sum_\gamma \frac{\gamma}{\alpha + \rho} \cdot \frac{1}{2^\alpha}\binom{\alpha}{\gamma} = \sum_\gamma \frac{\alpha}{\alpha + \rho} \cdot \frac{1}{2^\alpha}\binom{\alpha - 1}{\gamma - 1}$$
$$= \frac{\alpha}{2(\alpha + \rho)} \sum_\gamma \frac{1}{2^{\alpha - 1}}\binom{\alpha - 1}{\gamma - 1} = \frac{\alpha}{2(\alpha + \rho)}.$$

It is obvious that

$$P(G) = P(y) = P(Y) = P(g) = \frac{\alpha}{2(\alpha + \rho)}.$$

Since $P(r \mid \gamma, \upsilon, \rho) = \rho/(\alpha + \rho)$ does not depend on γ (and υ), it is also obvious that

$$P(r) = P(R) = \sum_\gamma P(r|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho) = \frac{\rho}{\alpha + \rho} \sum_\gamma P(\gamma, \upsilon, \rho) = \frac{\rho}{\alpha + \rho}.$$

One can easily see that this prior probability distribution over a future draw derived from the Bayesian model is equivalent to the heuristic model, in which prior "effective" urn content is $(\gamma, \upsilon, \rho) = (\alpha/2, \alpha/2, \rho)$.

Posterior. In the Bayesian model, second-order probability is updated following Bayes' rule. After an observed draw in green g,

$$P(\gamma,\upsilon,\rho|g)=\frac{P(g|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho)}{\sum_\gamma P(g|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho)}=\frac{P(g|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho)}{P(g)}=\frac{\gamma}{\alpha+\rho}\cdot\frac{1}{2^\alpha}\binom{\alpha}{\gamma}\cdot\frac{2(\alpha+\rho)}{\alpha}$$
$$=\frac{1}{2^{\alpha-1}}\binom{\alpha-1}{\gamma-1}.$$

Thus, posterior probability of a future draw in green after g is updated as

$$P(G|g)=\sum_\gamma P(G|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho|g)=\sum_\gamma\frac{\gamma}{\alpha+\rho}\cdot\frac{1}{2^{\alpha-1}}\binom{\alpha-1}{\gamma-1}$$
$$=\sum_\gamma\frac{\gamma-1}{\alpha+\rho}\cdot\frac{1}{2^{\alpha-1}}\binom{\alpha-1}{\gamma-1}+\sum_\gamma\frac{1}{\alpha+\rho}\cdot\frac{1}{2^{\alpha-1}}\binom{\alpha-1}{\gamma-1}$$
$$=\frac{\alpha-1}{2(\alpha+\rho)}\sum_\gamma\frac{1}{2^{\alpha-2}}\binom{\alpha-2}{\gamma-2}+\frac{1}{\alpha+\rho}\sum_\gamma\frac{1}{2^{\alpha-1}}\binom{\alpha-1}{\gamma-1}=\frac{\alpha-1}{2(\alpha+\rho)}+\frac{1}{\alpha+\rho}$$
$$=\frac{\alpha+1}{2(\alpha+\rho)}$$

(binomial theorem was used twice). Similarly,

$$P(Y|g)=\sum_\gamma P(Y|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho|g)=\sum_\gamma\frac{\alpha-\gamma}{\alpha+\rho}\cdot\frac{1}{2^{\alpha-1}}\binom{\alpha-1}{\gamma-1}=\frac{\alpha-1}{2(\alpha+\rho)}\sum_\gamma\frac{1}{2^{\alpha-2}}\binom{\alpha-2}{\gamma-1}$$
$$=\frac{\alpha-1}{2(\alpha+\rho)}.$$

$$P(R|g)=\sum_\gamma P(R|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho|g)=\frac{\rho}{\alpha+\rho}\sum_\gamma P(\gamma,\upsilon,\rho|g)=\frac{\rho}{\alpha+\rho}.$$

It is obvious that the posterior probability is equivalent to the heuristic account, in which posterior "effective" urn content after g is $(\gamma,\upsilon,\rho|g)=(\alpha/2+1/2\,,\alpha/2-1/2\,,\rho)$. The same applies to the posterior to y.

Lastly, a red (risky) draw r does not update the belief. In the Bayesian model, since $P(r\,|\,\gamma,\upsilon,\rho)=\rho/(\alpha+\rho)$ is constant,

$$P(\gamma,\upsilon,\rho|r)=\frac{P(r|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho)}{\sum_\gamma P(r|\gamma,\upsilon,\rho)\cdot P(\gamma,\upsilon,\rho)}=\frac{P(\gamma,\upsilon,\rho)}{\sum_\gamma P(\gamma,\upsilon,\rho)}=P(\gamma,\upsilon,\rho)$$

while in the heuristic account, $(\gamma,\upsilon,\rho|r)=(\alpha/2\,,\alpha/2\,,\rho)=(\gamma,\upsilon,\rho)$. Thus, $P(G|r)=P(G),P(Y|r)=P(Y),P(R|r)=P(R)$ in both models. ■

2. Uniform second-order prior

We also noted that similar behavioral and fMRI results could be yielded when we used uniform prior, another natural choice of second-order probability. In this case,

$$P(\gamma, \upsilon, \rho) = \frac{1}{\alpha + 1} \text{ for all } \gamma \text{ s.t. } 0 \leq \gamma \leq \alpha.$$

Prior probability in this model is actually identical to the model with binomial prior:

$$P(g) = P(G) = P(y) = P(Y) = \sum_{\gamma} P(g|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho) = \sum_{\gamma} \frac{\gamma}{\alpha + \rho} \cdot \frac{1}{\alpha + 1} = \frac{\alpha}{2(\alpha + \rho)}.$$

$$P(r) = P(R) = \sum_{\gamma} P(r|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho) = \frac{\rho}{\alpha + \rho} \sum_{\gamma} P(\gamma, \upsilon, \rho) = \frac{\rho}{\alpha + \rho}.$$

It is obvious that a risky draw r does not update the belief irrespective of the prior. Thus, the only difference between predictions of the binomial and uniform prior lies in posterior probability after ambiguous-color draws:

$$P(\gamma, \upsilon, \rho|g) = \frac{P(g|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho)}{\sum_{\gamma} P(g|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho)} = \frac{P(g|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho)}{P(g)} = \frac{\gamma}{\alpha + \rho} \cdot \frac{1}{\alpha + 1} \cdot \frac{2(\alpha + \rho)}{\alpha}$$
$$= \frac{2\gamma}{\alpha(\alpha + 1)}$$

$$P(G|g) = \sum_{\gamma} P(G|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho|g) = \sum_{\gamma} \frac{\gamma}{\alpha + \rho} \cdot \frac{2\gamma}{\alpha(\alpha + 1)} = \frac{2\alpha + 1}{3(\alpha + \rho)}$$

$$P(Y|g) = \sum_{\gamma} P(Y|\gamma, \upsilon, \rho) \cdot P(\gamma, \upsilon, \rho|g) = \sum_{\gamma} \frac{\alpha - \gamma}{\alpha + \rho} \cdot \frac{2\gamma}{\alpha(\alpha + 1)} = \frac{\alpha - 1}{3(\alpha + \rho)}$$

while $P(R|g)$ remains the same as the prior $P(R)$.

Note that equivalence to the heuristic account does not hold when the Bayesian model adopts uniform prior (or any non-binomial prior).

Behaviorally, this uniform-prior model successfully predicted values of ambiguous gambles after ambiguous-color draws ($p < .05$; Supplementary Fig. 2a), but we noted that it was outperformed by the original binomial (residual sum of squares, 109727 vs. 158925). Prediction of value updating based using the uniform model was also successful ($p < .05$), but was outperformed by binomial prior (residual sum of squares, 97861 vs. 147059). In fMRI analyses, activation maps under the uniform model were overall quite similar to the original results under the binomial model (Supplementary Fig. 2b).

Together, these show that our conclusions are overall robust with respect to the specifics of our original Bayesian modeling.